A Stochastic Fuel Switching Model for Electricity Prices[1]

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Abstract

In this paper a novel electricity price model is developed and applied. We reproduce the merit order of a thermal-dominated electricity system by establishing a non-linear dependency of wholesale electricity prices on the prices of fuels (coal and natural gas) and of CO_2 emission allowances. The coefficients are estimated using a Markov Switching Regression. This allows to study the nonlinear interaction between fuel and electricity prices. Consequently, this approach might prove valuable for cross-hedging positions in the fuel, electricity and emission spot markets. It is also of use for studying, to which decree electricity prices in different countries reflect fuel cost. Applying the model to the electricity markets of the UK and Germany we find that British electricity prices are quite well explained by short-run cost factors while German are less so.

Keywords: Electricity Prices, Markov Switching Models
JEL classification: L94, C22, D43

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.1 Introduction

Electricity markets differ from other commodity markets in various respects. Demand for electricity is inelastic in the short term, storing it is expensive, parts of the value chain exhibit characteristics of natural monopolies and reliable electricity supply has high macroeconomic importance. In addition, electricity might be produced by a set of different technologies each of which being characterised by different marginal cost.

With the emergence of electricity spot markets, the statistical behavior of the prices attracted the attention of speculators, arbitrageurs and risk managers, not least because future and option contracts are usually settled at the spot price. This opened a new field of study for financial mathematics, and it soon became obvious that electricity prices behave very differently from prices for physical commodities. Important findings include the observations that prices mostly contain no unit root (Lucia and Schwartz (2001), Worthington et al. (2005)); exhibit mean reverting behavior (Cartea and Figueroa (2005)); feature strong seasonalities; high volatility; fat tails; and long memory behavior (Haldrup and Nielsen (2006)). Sophisticated stochastic models for electricity prices were developed based on the observed characteristics.[3] Weather, fuel prices, emission allowance prices, available generation capacities, imports, transmission congestion, market structure and the national fuel mix are key explanatory variables for price behaviour. Those factors differ substantially between markets and thus electricity price models are often only suitable for a specific power exchange.

Due to certain specifics of electricity prices – e.g., strong heteroscedasticity and pronounced price spikes – Markov regime switching models have gained quite some popularity for modeling electricity prices.[4] These models allow to capture spikes in the price levels and variance regimes. Thus, price and volatility forecasts for electricity prices can be significantly improved. Furthermore, Markov regime switching models allow to make electricity prices in different regimes conditional to different linear combinations of fuel and emission allowance prices.[5] This makes it possible to econometrically represent the well known non-linear relationship between the different price series. Consequently, corresponding models can be

[3] Haldrup and Nielsen (2004) for example find a regime switching long memory model to adequately forecast spot prices in the Nordic market. Periodic heteroskedastic autoregressive fractionally integrated moving average model models were proposed by Carnero et al. (2007), and Cartea and Figueroa (2005) suggests jump diffusion models. Overviews of the relevant literature can be found in Bunn (2004), Knittel and Roberts (2004) and Skantze and Ilic (2001).

[4] For an overview see Janczura and Weron (2010).

used to better predict electricity prices, optimize portfolios consisting of electricity contracts, fuel contracts and emission allowances or compare the price formation in different countries. To our knowledge this paper is the application of a Markov regime switching model to represent the merit order. The next section introduces the data for the UK and Germany used in our model:. Section 3 presents the model, Section 4 presents the results and an interpretation, and Section 5 concludes.

.2 Data

The German and UK electricity systems are comparable in size (see Table 1). Conventional thermal power plants account for most of the electricity generation (62% in Germany and 78% in the UK). One obvious difference is that the UK does not use lignite for which it compensates by an increased share of natural gas.

Table 1: Gross electricity generation (2007)

	Germany	UK
Annual gross electricity generation	**637 TWh**	**396 TWh**
Coal	47%	35%
Oil	2%	1%
Gas	13%	42%
Nuclear	22%	16%
Renewables	15%	5%
Other	1%	1%
Source: Eurostat[6]		

Both countries' wholesale markets are particularly suited to our model. *First*, neither market is endowed with significant hydropower capacity. This is an advantage because our model is unable to reproduce the dynamic opportunity cost assessment required for analyzing the marginal cost of a hydro plant. *Second*, both markets provide reference prices. Hourly spot electricity prices for Germany are obtained from the EEX (prices are formed by day-ahead, two-side, one-shot, sealed-bid uniform-price auctions). By contrast, half-hourly spot prices at the UK Power Exchange (UKPX) are obtained in 48-hour continuous trading until a half-hour ahead of delivery.[7] However, both countries differ markedly in market structure and design. While the UK has two decades of experience with market opening and regulation, Germany only addressed sector reforms in the first part of this decade, and established a national

[5] In section 3 we explain that such a formulation essentially is a stylized representation of the fundamental merit order.
[6] The data were retrieved from Eurostat [http://epp.eurostat.ec.europa.eu].
[7] Most of the high-frequency price and volume data employed in this dissertation were not freely available, but could only be obtained upon request from the corresponding data providers. The usage rules did not allow a publication of the original data in this dissertation.

regulator in mid-2005. The four privately owned transmission system operators in Germany retain significant stakes in generation (together 80% of total capacity in 2009[8]) and distribution. The integration of the two major German players, E.on and RWE, and their natural gas affiliates enhances their dominance. The situation in the UK, on the other hand, is more balanced. The transmission system operator (TSO) is unbundled and national regulation is effective. The six biggest generation companies together were in 2007 responsible for around 69% of the electricity production.[9] Although they are integrated with electricity and gas suppliers, no one has a position comparable to the "big four" in Germany.

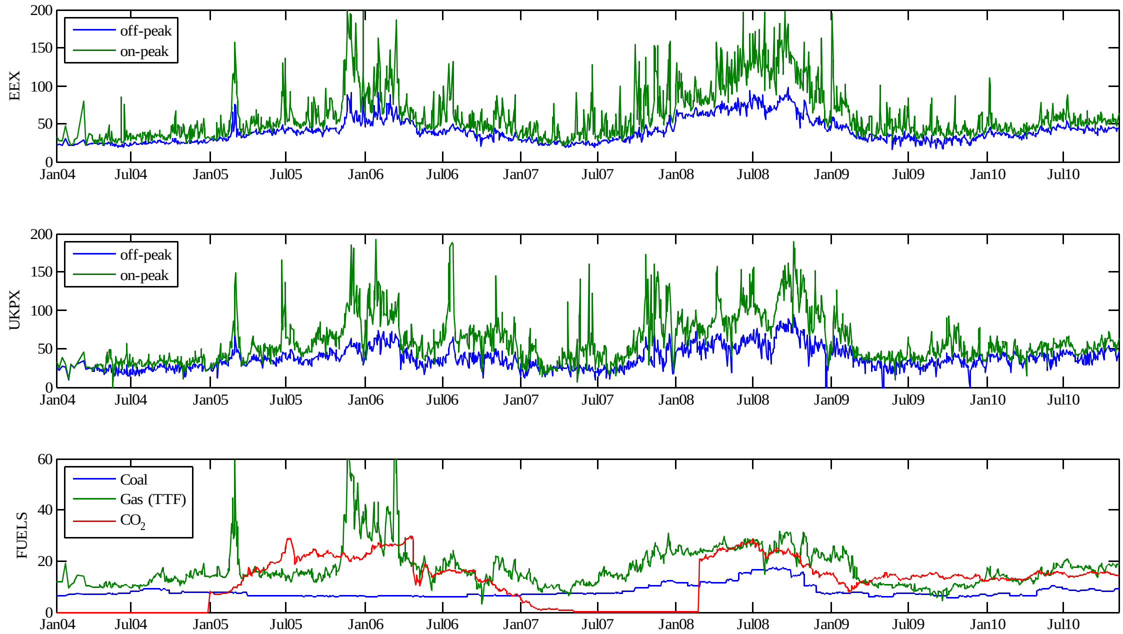
Table 2: Summary of the data sample (working days January 2004 - November 2010)

		Germany			United Kingdom		
	Unit	Source	Mean	Variance	Source	Mean	Variance
Electricity off-peak	€/MWh_{el}	EEX	38.0	348	UKPX	40.9	345
Electricity on-peak	€/MWh_{el}[10]	EEX	62.3	1682	UKPX	64.7	1764
Gas spot price	€/MWh_{th}	TTF	16.9	36	NBP	17.5	69
Coal spot price	€/ MWh_{th}	ARA	8.2	7	ARA	8.2	7
Emission allowance	€/EUA	EEX	12.1	79	EEX	12.1	79

Because our model is only meaningful in the short and medium run, we used daily price notations for all commodities. Since no daily German gas and coal prices were available, we use the respective values of the Dutch markets for natural gas (TTF[11]) and coal (ARA[12]).[13] This data has been obtained from Datastream[14]. The sample contains data from January 2004 to November 2010, eliminating weekends and holidays.[15] We converted the fuel prices into Euro per calorific value measured in Megawatt (€/MWh_{th}) to simplify interpretation. The respective data sources for the three commodities for Germany and the UK are summarized in Table 2. Figure 1 depicts the series of spot prices. [16]

8 Bundeskartellamt (2011, p18).
9 According to National Grid: British Energy (18%), E.on (13%), SSE (12%), RWE (10%), Edf(9%), DRAX (7%), Scottish Power (6%), International Power (5%) and Centrica (5%)
10 MWh_{el} stands for one Megawatt hour of electric energy.
11 TTF stands for Title Transfer Facility, a virtual trading point for natural gas in the Netherlands.
12 CIF ARA coal prices (CIF refers to Cost, Insurance and Freight and ARA refers to Amsterdam, Rotterdam, Antwerp).
13 It should be noted that gas and especially coal prices in Germany should exceed Dutch fuel prices by some constant because of transportation costs.
14 Thomson Datastream is a commercial financial statistical database.
15 This has the positive side-effect of significant reductions in weekly seasonalities.
16 Datastream derives the daily coal price notations by converting the monthly coal prices in dollars into Euros using the daily exchange rate. Thus, the increasing Dollar-Euro exchange rate limited the effect of rising coal prices for European coal consumers.

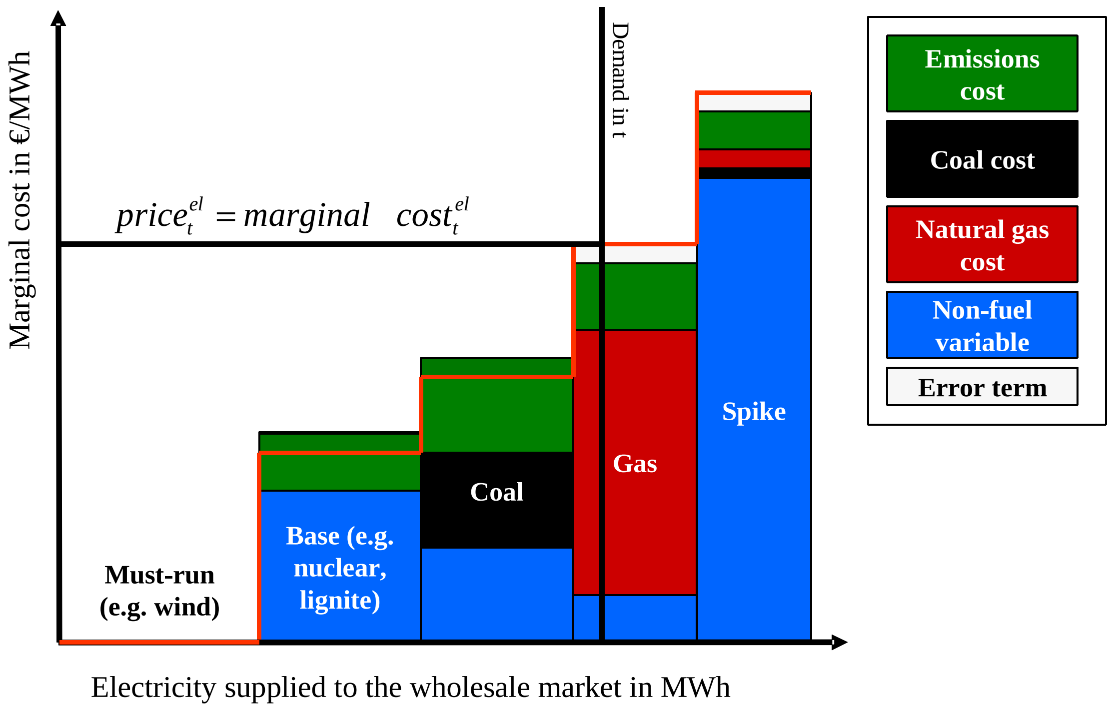
Figure 1: Development of the spot price series 2004-2010 (in €/MWh)



.3 Model

As mentioned, electricity is generated by a variety of technologies. Since the differences of marginal costs of power plants with the same technology are small compared to the cost difference between dissimilar technologies, we can approximate the marginal cost curve of the entire electricity system via a stepwise function (see Figure 2).[17]

Figure 2: Stylized example of the stepwise marginal cost function[18]



In detail, we can model the electricity price at time t as the marginal cost of the last required technology to meet demand. In the short run, the costs of a plant should correlate closely with its fuel and emission costs. Since the fuel efficiency of technologies changes rather slowly, fuel and emission costs are predominantly determined by the respective prices. Thus, we can create a time series model that endogenously infers the cost structures of each class of power plant and deduces which class of power plant is marginal at each point. This methodology requires data on fuel, emission and electricity prices only.

[17] Typical non-dispatchable, must-run generation includes wind, run-of-river hydro and combined heat and power plants (in winter).
[18] The error term might be positive or negative. Consequently the marginal cost might in some cases be lower (base and coal in the stylized representation) or higher (gas and spike in the stylized representation) than the sum of fuel, emission and other variable cost.

Generally, our model consists of two procedures: a routine that decides which class of power plants sets the price (i.e., is marginal) and a mechanism that reproduces the electricity price formation for each class. For each technology regime[19] $S_t = 1,2,...,m$ we assume the marginal costs at time $t \in 1 \ldots T$ to be the sum of a state dependent stochastic component and a state dependent weighted linear combination of the *k* explanatory variables. That is, the weighting vector β depends on the marginal technology in time t. Thus, the weighting vector specific for the technology state i (i.e., the vector that applies for all t where $S_t = i$) is denoted β_i. The state dependent stochastic component in time t for the technology state i is denoted $\varepsilon_{t,i}$. The set of explanatory variables stored in the k rows of the matrix $X_{1:T}$ may contain, for example, a constant, a time trend and different dummy variables, as well as gas, oil, coal and emission certificate prices.[20] Depending on the chosen explanatory variables and the technologies, the model can be written as

$$price_t^{el} = \begin{cases} \beta_1 \times X_t + \varepsilon_{t,1} & S_t = 1 \\ \beta_2 \times X_t + \varepsilon_{t,2} & S_t = 2 \\ \quad\vdots & \\ \beta_m \times X_t + \varepsilon_{t,m} & S_t = m \end{cases} \tag{1}$$

When the process that determines the marginal technology at time t is assumed to be Markovian[21] and $\varepsilon_{t,i}$ is assumed to be independent and identically normal distributed in each technology state i, (1) can be estimated using a Markov Switching Regression. We first convert the model into state space form with the states (or regimes) representing the different technologies. To make the model computable, we specify the transition matrix as $Prob(S_t = i \mid S_{t-1} = j) = p_{i,j}$, i.e. with time invariant exogenous switching probabilities.[22]

[19] The terms state, regime, technology and technology regime are used synonymously.

[20] The notion $1:T$, here and in the remainder of the paper, refers to the corresponding element at all discrete time points between $t = 1$ and $t = T$.

[21] A Markov process is characterized by the fact that the likelihood of a given future state, at any given moment, depends only on its present state, and not on any past states.

[22] Including demand and weather conditions in the switching probabilities could improve the estimation; modeling switching cost as threshold variables in the state-equation may make the estimates even more realistic. However, the problematic implementation is left to further research.

Thus, the model is fully described by

$$price_t^{el} = \beta_{S_t} \times X_t + \varepsilon_{t,S_t} \qquad (2a)$$

$$S_t = 1...m \qquad (2b)$$

$$Prob(S_t = i \mid S_{t-1} = j) = p_{i,j}\,, \quad \forall \quad 1 \le i, j \le m \qquad (3)$$

where X_t is the t^{th} column of the ($k \times T$) matrix $X_{1:T}$ of explanatory variables and β_{S_t} is the state dependent ($1 \times k$) row vector ($\beta_{S_t,1}, \beta_{S_t,2}, ... \beta_{S_t,k}$, where $\beta_{S_t,k}$ is the coefficient for the k^{th} explanatory variable).

The presented stylized merit order (see Figure 2) implies that there are only four types of power plants with different cost structures.[23] The marginal cost for each technology depends only on the fuel consumption, emissions and non-fuel variable costs. Thus, the explanatory variables are: a constant, the coal price, the gas price and the emission allowance price. We can impose certain zero restrictions on β_{S_t} because the marginal cost of coal plants should not depend on the gas price. The interpretation of the remaining coefficients is then straightforward. The constant represents the non-fuel variable cost of this type of plant. The fuel coefficient for the used fuel is the inverse of the heat rate of the plant (when electricity price and fuel price are both measured in the same unit, i.e. €/MWh). The coefficient for the emission certificate prices represents the amount of emissions per unit of electricity.[24] When interpreting the results, we must bear in mind that we do not address the endogeneity problem (i.e. we ignore the fact that gas and emission allowance prices also depend on electricity prices) and the number of states selection problem (i.e. we ignore that the “real” number of states might be different from our choice).

Our non-linear model makes it difficult to deduce theoretically the distribution of the parameters conditioned on the data. Thus, we rely on the approach by Schweri (2004) who proposes to address this issue by using the Gibbs sampling technique.[25] The general idea of Gibbs sampling is to repeatedly draw each parameter conditioned on the data and on all other parameters. This procedure is iterated many times, always conditioned on the latest draws of

[23] Must-run generation like wind and run-of-river hydro are not included since they can be considered as a reduction of net electricity demand.
[24] The units match accordingly: $€/MWh_{el} = €/MWh_{el} + MWh_{el}/MWh_{th} \times €/MWh_{th} + tCO_2/MWh_{el} \times €/tCO_2$
[25] See Krolzig (1997, p.148ff).

the other parameters. To estimate (2) and (3) via Gibbs sampling, the density function $g(\bullet)$ has to be separated. Schweri (2004) proposes the following dissection:

$$
\begin{aligned}
&g\left(S_{1:T}, \beta_{S_t}, \sigma_{S_t}, p_{i,j} \mid price_{1:T}^{el}, X_{1:T}\right) = \\
&g\left(\beta_{S_t}, \sigma_{S_t} \mid price_{1:T}^{el}, X_{1:T}, S_{1:T}\right) g\left(p_{i,j} \mid S_{1:T}\right) g\left(S_{1:T} \mid price_{1:T}^{el}, X_{1:T}\right)
\end{aligned} \tag{4}
$$

According to the dissection in (4) the distribution of the parameters conditioned on the data can be deduced using the four steps proposed in Schweri (2004, p.34ff):

1) Deduce $g\left(S_{1:T} \mid price_{1:T}^{el}, X_{1:T}\right)$ from $g\left(S_T \mid price_{1:T}^{el}, X_{1:T}\right)$ and $g\left(S_t \mid S_{t+1}, price_{1:t}^{el}, X_{1:T}\right)$ by backward iteration. Thereby $g\left(S_t \mid S_{t+1}, price_{1:t}^{el}, X_{1:T}\right)$ is calculated from $g\left(S_t \mid price_{1:t}^{el}, X_{1:T}\right)$ which is obtained from the Hamilton filter.

2) Draw the beta-distributed switching probabilities $p_{i,j}$ given $S_{1:T}$.

3) Draw the β_{S_T} given $price_{1:T}^{el}$, $X_{1:T}$, $S_{1:T}$ and σ_{S_t}.

4) Draw the σ_{S_t} given β_{S_T}, $S_{1:T}$, $price_{1:T}^{el}$ and $X_{1:T}$.

A detailed description of the steps and its technical implementation appears in Schweri (2004, p.33-54) who also provides the corresponding Matlab code.[26]

[26]As the author was unable to find any other implementation of a Bayesian Markov Switching Vectorautoregression in the literature he had solely relied on the excellently documented Matlab code provided in the diploma thesis by Urs Schweri (2004). The methodology used in this paper reproduces the regime switching model without Kalman Filtering of Schweri (2004, p.33-54). The author would like to thank Urs Schweri for his welcoming reply to arising questions.

Table 3: Dimension and notation of variables used in the Markov-Switching Model

Variable	Dimension	Explanation
$price_t^{el}$	*scalar*	Explained variable: electricity price series
t	*scalar*	time index
S_t	*scalar*	Indicator of the regime in time t
k	*scalar*	number of explanatory variables
m	*scalar*	number of states
T	*scalar*	termination date
$p_{i,j}$	*scalar*	probability to switch from state i to state j.
X_t	$k \times 1$	vector of explanatory variables at time t (constant, considered fuel and emission price series)
β_{S_t}	$1 \times k$	state dependent coefficient vector
ε_{t,S_t}	*scalar*	state dependent error term
σ_{S_t}	*scalar*	state dependent error variance

The described estimation strategy features certain drawbacks:

(1) Due to the definition of prior expectations, that (might) drive the posterior distribution of the parameters, the results are not purely data driven (Schweri, 2004, p.29)

(2) The assumptions on the distribution of error terms (independent, identical distributed) are not met for all time series.

(3) The results might depend on the selection of starting values.

(4) For small numbers of draws the results are not stable and there is no final certainty that the model converged to the global maximum. Thus, a high number of draws (100,000) is chosen which make the estimation computationally burdensome (about two hours for a single electricity price series of five years).

Despite these caveats, the presented estimation strategy is well suited to estimate the described model. As discussed in Krolzig (1997, p.175) in contrast to the Expectation Maximization approach the Gibbs sampling approach provides the posterior distribution of the parameters. Furthermore, it allows the inclusion of prior knowledge which is essential for our modeling strategy (see next section).

.4 Results

(a) Estimation Results

To estimate (2) & (3), a sensible choice of the dependent variable (i.e. the electricity price series) is crucial. As demand is highly volatile throughout the day, we may expect that up to five regime switches (nuclear->coal->gas->coal->nuclear) occur every day. Using a continuous hour-by-hour series is inadequate because regime persistency ($p_{i,i} >> p_{i,j}$) is required for stable estimates. Thus, it is preferable to divide the continuous series into 24 day-by-day series, each of which represents one hour. However, we note that estimating (2) & (3) for 24 (or even 48) series is impractical because of the similarity of some series (e.g. 3rd and 4th hour data), and the estimation procedure is computationally burdensome. We can reduce the 24 or 48 hourly price series to two series and still retain most information by drawing on a weighted average of on-peak (8am-8pm) and off-peak (8pm-8am) prices. We obtain the optimal weighting vector (in terms of variance explained) by Principal Components Analysis (PCA).[27] Then we exclude dates with electricity prices above 200€/MWh because such extreme price spikes could distort our analysis and cannot be explained by fuel cost fundamentals.[28]

We estimate (2) & (3) for the off-peak and on-peak series for the German (EEX) and the British (UKPX) markets. In all four cases (EEX off-peak, EEX on-peak, UKPX off-peak and UKPX on-peak), we apply a model in which spot electricity prices are explained by spot gas prices, spot coal prices and the respective emission allowance price. We omit oil prices and a trend after our initial estimations have suggested that they are not significant for any state. Variance and all β coefficients are selected to be state dependent.[29] To capture the effect that

[27] Principal Component Analysis was developed to find those linear combinations of the elements of the columns of a data matrix that explain the majority of the variance of the data. A standardized linear combination is a weighted average ($\delta' X$) of the columns of X where δ is a vector of length one. Maximizing the variance of $\delta' X$ leads to the choice $\delta = \gamma_1$, the eigenvector corresponding to the largest eigenvalue λ_1 of the Covariance Matrix. This is a projection of X into the one-dimensional space, where the components of X are weighted by the elements of γ_1. $Y_1 = \lambda_1'(X - \mu)$ is called the first PC. This projection can be generalized to the second, third, and pth PCs by using the second, third, and pth largest eigenvalues and their corresponding eigenvectors. For the technical details see, for example, Haerdle and Simar (2003).

[28] Even burning expensive oil (95€/barrel) in an inefficient generator (heat rate of 30%) would only justify marginal cost of below 200€/MWh_{el} (0.625 barrel/MWh_{th} x 3.3 MWh_{th}/MWh_{el} x 95 €/barrel). For the modeling of electricity price spikes, see Lang and Schwarz (2007).

[29] Note that state dependent variance is straightforward since high electricity price regimes are characterized by higher variance.

switching from one marginal technology to another only occurs when demand or supply conditions change significantly, we predefine some persistency.[30]

Choosing the number of states is based on goodness-of-fit; interpretability with respect to the stylized merit order; and comparability. We measure goodness-of-fit using the Schwartz information criterion, which suggests that either three or four regimes are appropriate, depending on the case.[31] The assumed stylized merit order suggests that there are three regimes in off-peak (base, coal, gas) and three regimes in on-peak (coal, gas, spike). For ease of presentation and comparability, we use the three-state specification.

Table 4: Results of the switching regression with non-informative priors

	freq	$\beta_{Constant}$	β_{Coal}	β_{Gas}	β_{CO2}	Mean	σ^2
Germany off-peak (R^2=81%)							
State1	42%	**-2,40**	**1,54**	**0,94**	**0,56**	32,3	25
State2	44%	-0,50	**1,68**	**1,20**	**0,67**	39,8	15
State3	14%	**27,10**	-1,09	**1,55**	0,02	49,3	175
Germany on-peak (R^2=81%)							
State1	51%	**2,10**	**1,73**	**1,28**	**0,74**	41,1	45
State2	33%	**18,30**	**1,31**	**1,76**	**0,48**	67,0	90
State3	16%	**61,50**	0,95	**1,54**	0,30	110,0	815
UK off-peak (R^2=93%)							
State1	78%	1,20	**1,33**	**1,13**	**0,69**	34,4	9
State2	21%	**10,60**	**2,55**	**0,75**	0,32	65,0	64
State3	1%	**98,70**	**4,76**	1,54	**-5,10**	80,1	36
UK on-peak (R^2=88%)							
State1	62%	0,50	**2,02**	**1,38**	**0,82**	44,1	26
State2	23%	**8,10**	**2,67**	**1,66**	**0,75**	71,0	94
State3	15%	**52,30**	**2,02**	**1,56**	0,34	121,6	590
Bold empirical parameters are significantly different from zero. Freq = relative frequency that state *i* had the highest probability.							

The model is first estimated imposing (almost) no prior information on the parameters, switching probabilities and variances. Therefore, prior mean and starting values of the model parameters are set according to Table 8 (see Appendix). The estimation results for the three-

[30] The probability of remaining in the current state was set to 0.67 whereas the probability of switching to another state was adjusted to 0.16. Giving the prior a modest variance of approximately 0.1 implies that the beta-distribution of the p_{ij} - values is set to $u_1 = 2$ and $u_2 = 1$ on the main diagonal and $u_1 = 1$ and $u_2 = 6$ beyond the main diagonal.

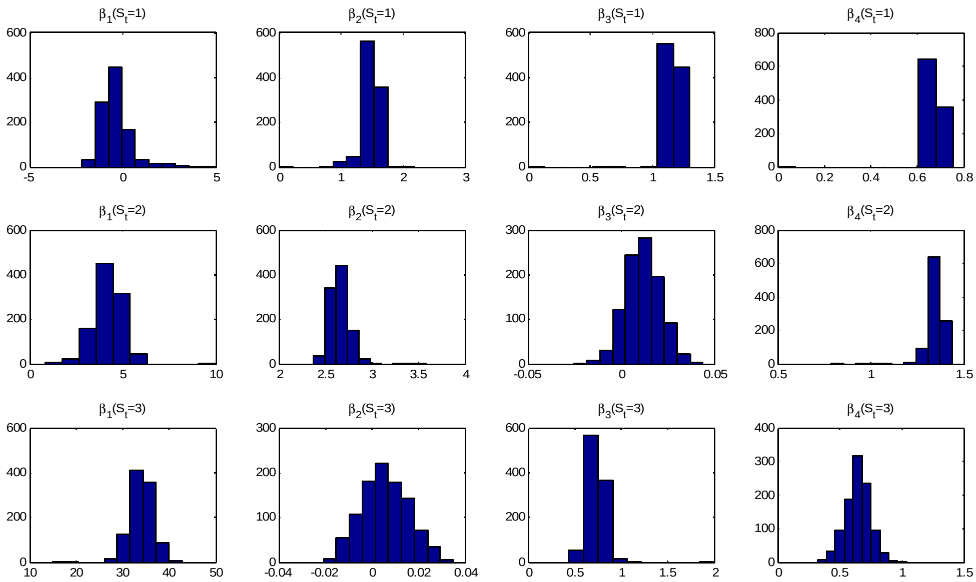
[31] The Schwartz Information Criterion has been calculated for each case for one to four regimes using a model specification with non-informative priors for the entire sample. While the Schwartz Information Criterion favors a three-regime specification for the UK off-peak case, a four-regime specification is preferred for all other cases. This reflects the higher diversity of the German off-peak generation structure and must be borne in mind when interpreting the results.

state model with non-informative priors (see Table 4) suggest that the regime-switching model adequately captures the electricity prices. *First*, the R^2 is above 80% for all series. *Second*, the model performs significantly better than the single- state model. *Third*, almost all empirical parameters are significantly different from zero. Despite this evidence, in other respects the model with non-informative prior's deviates markedly from the assumed stylized merit order as certain states overlap[32] and cannot clearly be attributed to the assumed technologies. Moreover, the occurrence of negative parameters is not explained.

Using informative priors, it is possible to induce model outcomes that are plausible with respect to the stylized merit order. In all four cases (EEX off-peak, EEX on-peak, UKPX off-peak and UKPX on-peak), certain parameters are constrained to zero by applying tight prior distributions with mean zero.[33] Setting the mean and variance for the prior distribution of the parameters as well as the starting values according to Table 9 (see Appendix), the model is estimated using the described procedure.[34] This selection ensures that in each case three technology regimes (off-peak: base, coal and gas; on-peak: coal, gas and spike) exist that can be clearly distinguished. The coal and gas price parameter priors, for example, imply that each fuel is only significant in the corresponding regime.

[32] In three of the four cases, the coefficients of at least two regimes cannot clearly be distinguished because the 95% confidence intervals for all of their coefficients overlap. This occurs for Germany off-peak (State1~State2) and UK peak (State1~State2).
[33] In each of the steps, the posterior distribution $p(\theta|y)$ is given by the likelihood function $L(\theta|y)$ times the prior distribution $g(\theta)$: $p(\theta|y)= g(\theta) \times L(\theta|y)$.
[34] Due to the identification restriction, the sorting of the no-fuel state is crucial. Setting it as the first state implies that it has the lowest constant of all states, resulting in a baseload state.

Figure 3: Posterior parameter density for the UK off-peak case (informative priors)



The estimation results (see Table 5) indicate a good fit and the estimated regime characteristics allow us to make a straightforward interpretation. *First*, each state can meaningfully be attributed to a unique technology. *Second*, the estimated parameters are in an intuitive order of magnitude. In all four cases, we note that the coal coefficient in the coal state is always larger than the gas coefficient in the gas state, and the emission allowance price has a stronger influence on the coal state. *Third,* almost all posterior parameter densities have a single maximum and are approximately normally distributed. This is illustrated in the UK off-peak example in Figure 3 where none of the posterior parameter distributions has two maximas.[35]

Each of the four technology regimes (base, coal, gas and spike) feature unique characteristics. In the *base regime*, electricity prices depend on both fuel prices and emission allowance prices. Whether the gas and coal price dependence can be explained by ramping and balancing costs that figure into the marginal cost of typical baseload plants (nuclear, wind, lignite) or whether the dependence is due to endogeneity (e.g., baseload electricity as a substitute for coal and gas) or another source of correlation between different energy sources cannot be determined. In the UK and Germany the *coal states* feature highly significant influences of coal prices, insignificant influences of gas prices and highly significant influences of emission allowance prices. As expected the empirical emission allowance price

[35] The results for all other cases may be obtained from the author upon request.

parameter for the *coal state* is higher than that for the *gas state*. In the *gas state,* all but the coal price coefficients are significantly positive. The empirical gas price parameters vary between 0.73 and 2.14, and the empirical emission allowance price parameters between 0.46 and 0.76. Finally the *spike state* is characterized by high prices and high variance.

Table 5: Results of the switching regression with <u>informative</u> priors

	freq	$\beta_{Constant}$	β_{Coal}	β_{Gas}	β_{CO2}	Mean	σ^2
Germany off-peak (R^2=83%)							
State "Base"	37%	**1,50**	**0,80**	**1,04**	**0,55**	34,0	51
State "Coal"	30%	**4,90**	**2,62**	0,01	**0,90**	31,6	11
State "Gas"	33%	**7,90**	0,00	**1,82**	**0,46**	48,3	47
Germany on-peak (R^2=82%)							
State "Coal"	47%	1,30	**3,70**	0,01	**1,10**	40,4	44
State "Gas"	40%	**19,30**	0,00	**2,14**	**0,60**	65,1	137
State "Spike"	13%	**96,70**	**1,02**	0,34	0,13	118,3	709
UK off-peak (R^2=91%)							
State "Base"	67%	-0,40	**1,49**	**1,16**	**0,67**	36,1	9
State "Coal"	21%	**4,20**	**2,65**	0,01	**1,35**	40,2	5
State "Gas"	12%	**34,10**	0,01	**0,73**	**0,65**	68,8	148
UK on-peak (R^2=83%)							
State "Coal"	36%	1,80	**3,66**	0,01	**1,20**	41,1	19
State "Gas"	45%	**16,40**	0,00	**1,82**	**0,76**	55,9	105
State "Spike"	19%	**87,60**	-0,52	**0,77**	**0,97**	114,0	678
Bold empirical parameters are significantly different from zero.							

However, there are some limitations to the results. *First,* it is difficult to explain that despite the straightforward identification of technology regimes, the cost structures of the technologies are unstable across countries and load periods. For example, the empirical parameter for gas in the UK gas states (off-peak: 0.73, peak: 1.82) do not overlap. *Second*, some coefficients are far "off" their expected values. For example, the inverse heat rate of a gas- fired plant should be somewhere around 2.5, but the estimated values are significantly smaller. *Third,* the assumption of normality for the residuals must be rejected for some regimes[36].

There are two potential explanations for the deviations of the estimation results from expectations: either the model is misspecified with respect to the real marginal cost of

[36] The different tests lead to very different results. While the Kolmogorov-Smirnov test only rejects the normality hypothesis for the gas state in the UK on-peak model at the 5% significance level, the Jarque-Bera-Test rejects normality for nine of the 12 cases. This indicates, that the residuals might be close to normal at the centre of the distribution while they feature heavy tails.

electricity production, and/or the underlying assumption that electricity prices are based on marginal cost is incorrect. While the first explanation probably holds to some degree,[37] there are reasons to believe that the second explanation is also plausible. Since the cost structure of a national power generation system is rather stable, intertemporal and international comparison of the model outcomes allows us to track the differences in the deviations of electricity prices from marginal cost.

(b) Intertemporal and international comparison of price formation

The goodness-of-fit of our model is better in the UK in off-peak (see Table 6). Using the Wilcoxon rank sum test[38], we find that the median absolute errors are significantly larger in the German case.

Table 6: Goodness-of-fit (R^2) of the regime switching model with informative priors

	Germany	UK	Wilcoxon rank sum test results[39]
on-peak	82%	83%	2,505,203
off-peak	83%	91%	2,686,014***

Figure 4 depicts the marginal state for every point in time as estimated in the model with informative priors for the UK off-peak case. What is striking about this example is the dominance of the gas regime in the years 2004, 2007 and 2010. Thus we are able to track the fuel switching due to high emission certificate prices, lower baseload generation margins produced by increasing baseload demand and/or decreasing baseload generation capacities.

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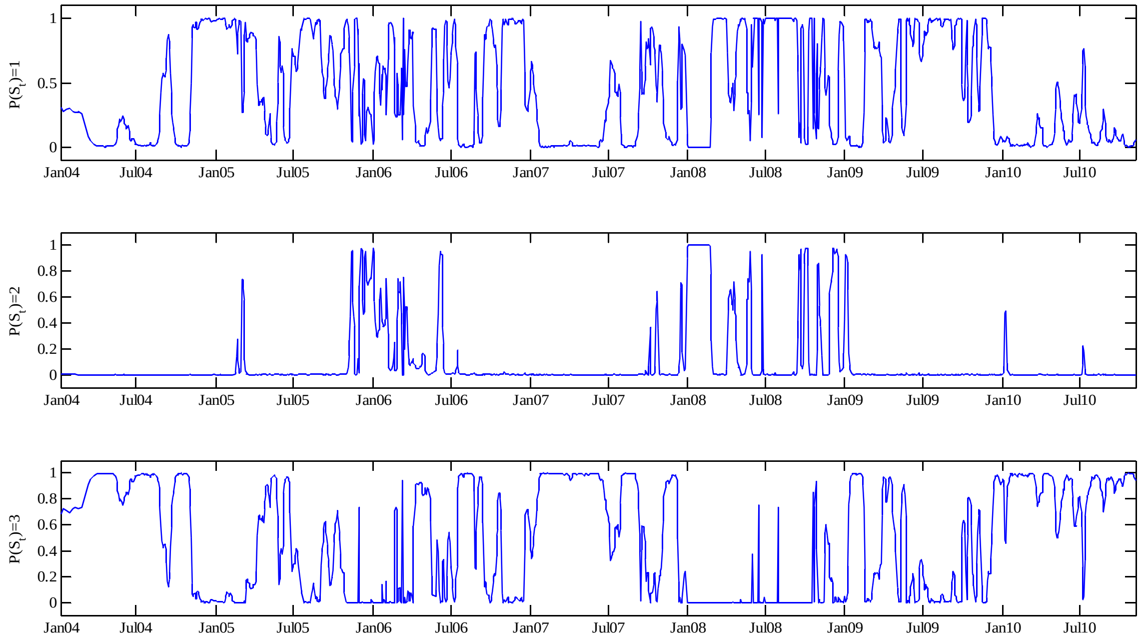
[37] One cannot expect that a stochastic model with a very parsimonious specification can completely track the marginal cost of a complex electricity system. It is likely that increasing the number of technologies (i.e., states) and including more data (e.g. demand) could improve the outcomes.

[38] For details on the Wilcoxon rank sum test see Freund and Wilson (2002, p.588ff).

[39] We test the null hypothesis that the median of the absolute value of the residuals for the German and the British model are identical. The three stars indicate that the null hypothesis is rejected on the 99% significance interval. Note that the variance of both price series have been approximately similar.

Figure 4: Regime probabilities for the UK off-peak case (informative priors)



.5 Conclusion

The presented model is able to reproduce the nonlinear interaction of fuel, emission allowance and electricity prices in the UK and Germany. As multi-commodity options are quite common in electricity markets (e.g., dark spread, spark spread and clean spread options) the ability of this modeling strategy to statistically represent the co-movements of these commodities could prove helpful for evaluating corresponding options.

In the context of analyzing strategic behavior in the power sector, the responsiveness of electricity prices on fuel price shocks is an indicator that can be easily compared across countries. We deduce from our analysis, that the UK off-peak prices are much closer linked to fuel prices (91% of the variance explained) than the German off-peak prices (83%). However, the different reasons for this divergence cannot easily be separated. Thus, the higher shares of renewable electricity generation and lignite in Germany, the stronger integration of Germany into the European market and a higher market concentration in Germany might be among the reasons for the observed differences.

Of course more research is needed and model extensions are desirable. For example, making regime switching conditional to electricity demand could improve forecasts.[40] And, making electricity prices in each regime mean reverting (and thus the process autoregressive) – and each regimes mean conditional on fuel prices could further enhance performance.

[40] Janczura and Weron (2010).

.6 Appendix

Table 7: Result of the ordinary least square model for the electricity price

	UK off-peak (R^2=89%)		UK on-peak (R^2=73%)		German off-peak (R^2=68%)		German on-peak (R^2=57%)	
	beta	tstat	beta	tstat	beta	tstat	beta	tstat
β_{const}	**-5.3**	-11.2	**-25.1**	-16.5	1.1	1.6	**-10.3**	-5.8
β_{coal}	**2.8**	45.5	**4.4**	24.3	**1.2**	12.1	**1.6**	7.0
β_{gas}	**1.0**	52.5	**2.4**	38.2	**1.3**	27.6	**2.9**	26.3
β_{C02}	**0.7**	40.4	**0.8**	14.5	**0.5**	19.7	**0.7**	10.3
Bold coefficients are significantly different from zero.								

Table 8: Prior mean and starting values of the model with non-informative priors

	State 1	State 2	State 3
β_{const}	5	10	15
β_{coal}	1	1	1
β_{gas}	1	1	1
β_{C02}	1	1	1

Table 9: Prior mean (prior variance) and starting values of the model with informative priors

	off-peak			on-peak		
	Base	Coal	Natural Gas	Coal	Natural Gas	Spike
β_{const}	5 (10)	10 (10)	15 (10)	5 (10)	10 (10)	100 (10)
β_{coal}	0 (1)	3 (1)	0 (.0001)	3 (1)	0 (.0001)	0 (1)
β_{gas}	0 (1)	0 (.0001)	2 (1)	0 (.0001)	2 (1)	0 (1)
β_{C02}	0 (1)	1 (1)	1 (1)	1 (1)	1 (1)	0 (1)

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